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Escaping Solitons from a Trapping Potential

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Solitons propagating within a confining potential undergo momentum-dependent scattering and eventually escape for large excitations. We experimentally highlight this phenomenon in the presence of a nonperturbative nonlinear response using self-confined light beams in a reorientational medium.

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Spatial solitary waves or solitons are known to occur in several areas of physics including fluids, plasmas, biology, matter waves, and optics [1,2]. They are often considered ubiquitous and share a number of fundamental properties relying on the combination of a nonlinear response and the natural tendency of a wave packet to spread as it propagates. Spatial solitons in optics (bright transversely self-localized light beams) are recognized as fundamental nonlinear electromagnetic wave objects with potential applications to all-optical signal processing [3,4] and have been explored in several materials and configurations for signal readdressing [5–11]. Owing to their self-guided nature and *robust* particlelike behavior, in the presence of dielectric inhomogeneities, soliton dynamics embraces various phenomena, from oscillations and breathing [12–14] to refraction and reflection, [5,15,16], steering [17,18], as well as confinement near a surface [19,20]. As predicted in the early days of soliton optics [21–23], they undergo scattering at interfaces and are known to oscillate within a wide (preestablished) waveguide, with a period depending on launch conditions, [13,14,16,18,24]. Solitons are also expected to escape a trapping potential when their effective kinetic energy becomes comparable with the barrier depth [23,25], i.e., when the nonlinear disturbance induces a transverse acceleration large enough to overcome linear confinement. Such wealth of effects extends beyond optics [25–28] including, among others, Bose Einstein condensates [29].

In spite of the predictions and interest in nonlinear scattering and trapping [30,31], soliton escape from a potential well was not reported to date mainly because of limitations in propagation length and nonlinearity. Soliton tunneling phenomena were recently reported in Ref. [32]. In this Letter, we investigate the interaction of spatial solitons with an externally defined potential and demonstrate for the first time soliton escape above a certain level of excitation. To this aim, we exploit the giant nonlinear optical response of nematic liquid crystals (NLC).

Nematic liquid crystals are molecular dielectrics with properties intermediate between solids and liquids, with a high degree of orientational order resulting in large optical

birefringence [33]. Electric fields at optical frequencies can reorient the optic axis (“director”) towards the polarization direction, inducing a Coulombian torque counteracted by intermolecular (elastic) forces. As a result, their nonlinear response is highly nonlocal, i.e., the index perturbation extends well beyond the excitation region and supports stable and robust (2D + 1) spatial solitons [34]. Moreover, NLC can undergo all-optical index changes which, at milliwatt power levels, are comparable to the size of their birefringence.

We consider a cell layout as in Fig. 1: two parallel glass slides confine a layer of nematic liquid crystals and provide anchoring for molecular alignment in the plane $\hat{t}\hat{p}$, being \hat{p} normal to the input facet. The optic axis (molecular director) \hat{n} forms an angle θ with the plane $\hat{t}\hat{p}$ and ϕ with the planar interface parallel to $\hat{x}\hat{t}$ and sealing the sample at the entrance [see Figs. 1(a) and 1(b)]. Voltages can be applied via thin film electrodes and alter the molecular alignment (and optical properties) in the bulk NLC. The front electrode is split into two by an L -wide gap along \hat{p} ; hence, distinct potentials can be applied to each of them with respect to the ground plane at the bottom [see Fig. 1(a)]. The two corresponding NLC regions, labeled 1 and 2, are ideally separated by the plane $\hat{x}\hat{p}$. The director distribution is described by $\hat{n}(x, t, p) = \hat{n}(\sin\theta, \cos\theta\cos\phi, \cos\theta\sin\phi)$ and, for a constant director alignment along

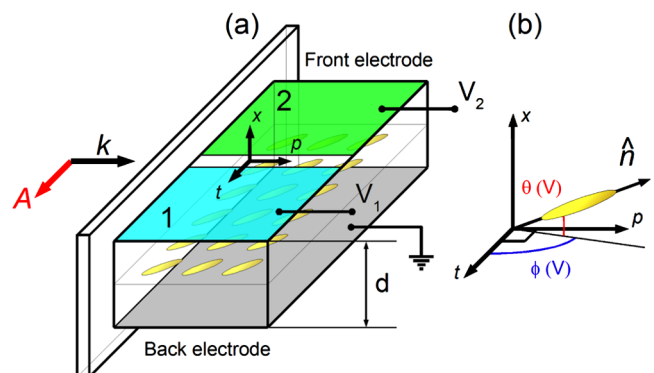


FIG. 1 (color online). (a) Sketch of the cell. (b) Molecular director (optic axis) and coordinate system.

\hat{p} , all-optical reorientation is governed by the coupled equations [5]:

$$2\epsilon_a E_x E_t \cos 2\theta \cos \phi + \epsilon_a \sin 2\theta (E_x^2 - E_t^2 \cos^2 \phi) + K \sin 2\theta \left[\left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} \right)^2 \right] + 2K \nabla^2 \theta = 0 \quad (1)$$

$$\epsilon_a E_x E_t \sin 2\theta \sin \phi + \epsilon_a E_t^2 \cos^2 \theta \sin 2\phi + \cos \theta \left[4K \sin \theta \left(\frac{\partial \theta}{\partial x} \frac{\partial \phi}{\partial x} + \frac{\partial \theta}{\partial t} \frac{\partial \phi}{\partial t} \right) - 2K \nabla^2 \phi \right] = 0 \quad (2)$$

being E_x and E_t the electric field components of the optical wave, $\epsilon_a = \epsilon_{\parallel} - \epsilon_{\perp}$ the low-frequency dielectric anisotropy, and K the NLC Frank elastic strength in the one constant approximation [33]. One of the crucial aspects of solitons in an NLC cell with nonuniform (electrically modified) director alignment is the out-of-plane reorientation of the optic axis [$\theta(V) > 0$], the latter implying out-of-plane walk-off of the extraordinarily polarized wave component: for an optical wave vector \mathbf{k} lying in the midplane $\hat{t}\hat{p}$ ($x = 0$), the energy flow (Poynting vector), i.e., the beam trajectory, moves towards one of the NLC-glass boundaries. As this can be hardly compensated in NLC, its consequence is out-of-plane soliton dynamics [35]. However, if the trapping potential is defined by altering exclusively the in-plane reorientation ϕ , for an optical wave vector in the cell midplane, the Poynting vector does not possess x -components. We adopted the geometry in Fig. 2: the two top contacts are biased by phase-locked π -shifted sinusoidal voltages at 1 kHz, such that their relative root-mean-square (RMS) potential is twice larger than the voltage between each front electrode and the ground plane. Therefore, underneath the electrode gap, the electric field has a sizable t -component and can induce a significant change in ϕ .

By numerically integrating the system (1) and (2), we evaluated the director distribution: Fig. 3 displays the distributions of θ and ϕ in a cell filled with E7, a commercial NLC ($\epsilon_{\parallel} = 19\epsilon_0$, $\epsilon_{\perp} = 10\epsilon_0$, $K = 1.2 \cdot 10^{-12}$ N), with $\phi(V=0) = \phi_0 = 60^\circ$, $V_1 = 0.55$ V, and $V_2 =$

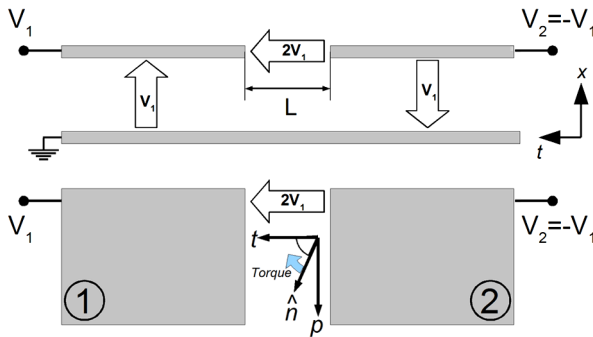


FIG. 2 (color online). Applied polarization: the potential difference between the two front electrodes is twice larger than between each front electrode and the ground plane. The corresponding torque can rotate the molecules in the plane $\hat{t}\hat{p}$.

-0.55 V (the sign in front of the RMS value emphasizes the π phase-shift), an interelectrode gap $L = 50 \mu\text{m}$ and a cell (NLC) thickness $d = 100 \mu\text{m}$. We set $\theta(V=0) = \theta_0 = 3^\circ$ as the sample was realized with a small anchoring pretilt in order to avoid disclinations [33]. Clearly, the low-frequency field induces a few degrees of distortion in both θ and ϕ , with a minimum ϕ in the middle of the electrode gap.

The nematic E7 is a positive uniaxial with $n_{\parallel} = 1.6954$ and $n_{\perp} = 1.5038$ at $\lambda = 1064$ nm; hence, for an extraordinarily polarized (“ e ” wave) optical field, the index increases monotonically with angle Θ between the wave vector \mathbf{k} and \hat{n} , being $\Theta = \cos^{-1}[\cos(\theta) \sin(\phi)]$ for \mathbf{k} at small angles with respect to \hat{p} [35]. After a Taylor expansion of Θ , it is easy to verify that the index perturbation is essentially governed by ϕ ; thereby, the relatively wide waveguide (comparable to L) just below the gap entails negligible out-of-plane walk-off for e -waves.

An equivalent particle model for the soliton-waveguide interaction can be derived from a generalized nonlinear Schroedinger equation (NLS) [12],

$$2ik_v n_0 \frac{\partial A}{\partial p} + \nabla^2 A - M[x, |A|^2]A = 0 \quad (3)$$

being $k_v = 2\pi/\lambda$ the wave number in vacuum, $M = -k_v^2[n(\mathbf{r}, |A|^2)^2 - n_0^2]$, $\mathbf{r} = x\hat{x} + t\hat{t}$, n_0 the unperturbed refractive index for beams propagating close to \hat{p} , $n(\mathbf{r}, |A|^2)$ the index perturbed by voltage bias and optical field amplitude A . Naming $P_A = \iint_{-\infty}^{\infty} |A|^2 dx dt$, we can describe the trajectory of the beam center $\langle \mathbf{r} \rangle = \iint_{-\infty}^{\infty} \mathbf{r} A A^* dx dt / P_A$ by deriving its transverse speed $\mathbf{v} = \partial \langle \mathbf{r} \rangle / \partial p$ and acceleration $\mathbf{a} = \partial^2 \langle \mathbf{r} \rangle / \partial p^2$,

$$\mathbf{v} = \frac{1}{2ik_v n_0 P_A} \iint_{-\infty}^{\infty} [(\nabla_{xt} A) A^* + (\nabla_{xt} A^*) A] dx dt \quad (4)$$

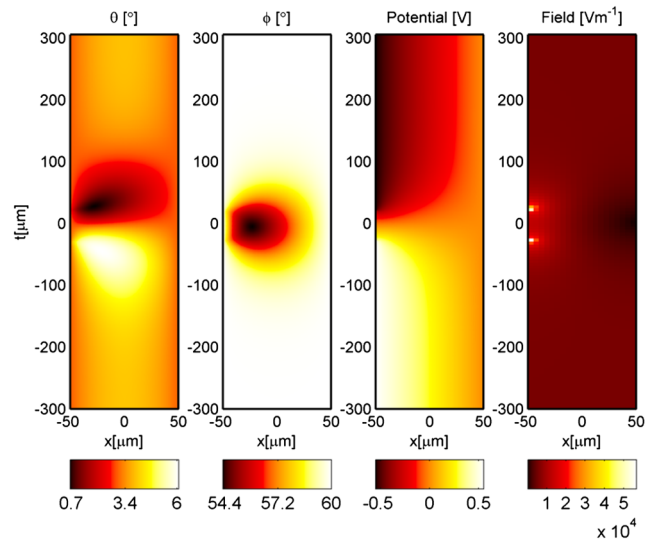


FIG. 3 (color online). Distribution of θ , ϕ , low-frequency potential and electric field distribution for $V_1 = 0.55$ V and $V_2 = -0.55$ V in a cell with E7.

$$\mathbf{a} = -\frac{1}{2k_v^2 n_0^2 P_A} \iint_{-\infty}^{\infty} AA^* (\nabla_{x_l} M) dx dt, \quad (5)$$

i.e., applying the Ehrenfest's theorem to the NLS. Considering trajectories in the plane $\hat{t} \hat{p}$ and $\langle t \rangle = \langle \mathbf{r} \rangle \hat{t}$, if $n^2 - n_0^2$ is small, we write $n(t, |A|^2)^2 - n_0^2 \approx 2n_0 n_l(t) + 2n_0 n_{nl}(t, |A|^2)$, where $n_l(t)$ represents the bias-induced waveguide and $n_{nl}(t, |A|^2)$ the nonlinear perturbation. Defining a scaled $\varphi = A/P_A$, then $|\varphi|^2 = X(x)T(t - \langle t \rangle)$ describes a cylindric soliton centered in $\langle t \rangle$ and $\iint_{-\infty}^{\infty} X(x) dx = \iint_{-\infty}^{\infty} T(t) dt = 1$. The t -component of the acceleration is

$$a = \frac{1}{n_0} \int_{-\infty}^{\infty} T(t) \left[\frac{dn_l}{dt} + \int_{-\infty}^{\infty} X(x) \frac{\partial n_{nl}(t, |A|^2)}{\partial t} dx \right] dt. \quad (6)$$

If $n_{nl}(t, |A|^2)$ conserves the symmetry of $|A|^2$, then the mean nonlinear contribution to the transverse acceleration is null [24]. In our case, symmetry is broken by the angle dependence in the effective nonlinear response [36], the latter being lower within the waveguide. We can simply write $n_{nl}(t, |A|^2) = n_2(t) P_A (G \otimes |T|^2)$ (\otimes indicates a convolution integral) with $n_2(t)$ the position dependent nonlinearity and $G = 1/(\sqrt{\pi} w_{nl}) \exp[-(t/w_{nl})^2]$ the nonlocal response, w_{nl} being the range of nonlocality. It follows that, for solitons of spotsize w_s significantly smaller than w_{nl} , their profile has a negligible effect on the transverse dynamics.

Figure 4 shows the calculated beam trajectories and their concavity for solitons propagating in the midplane $x = 0$, having evaluated the linear index distribution from system (1) and (2). From Ref. [36], we estimated $n_2(t)$ with $|n_2(0) - n_2(\infty)| \approx 3 \times 10^{-13} \text{ m}^2/\text{V}^2$ and assumed a highly nonlocal regime with $w_{nl} = 30 \mu\text{m} \gg w_s$ [34]. As foreseen, the equivalent particle escapes the potential owing to a nonlinear acceleration. The concavity of the trajectory $\partial^2 \langle t \rangle / \partial p^2$ moves towards zero as the soliton escapes from the linear potential.

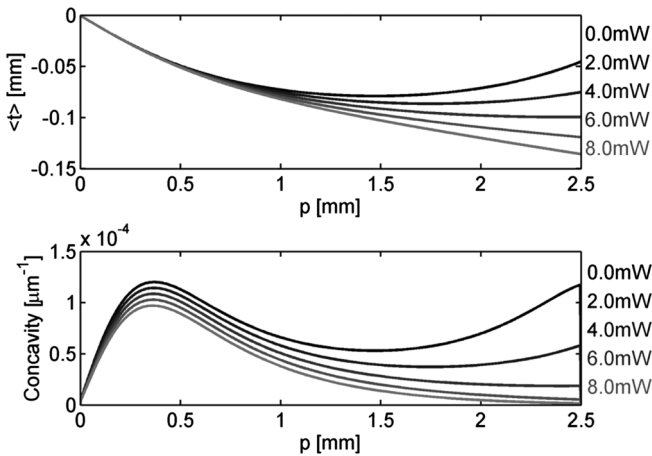


FIG. 4. Particle model: (top) calculated trajectory for various excitations; (bottom) the corresponding concavity $\partial^2 \langle t \rangle / \partial p^2$ of the beam path approaches zero as the power increases (black to gray lines).

We experimentally excited a soliton by launching a Gaussian beam with a diffraction length $L_d = 100 \mu\text{m}$ from a Nd:YAG laser operating at $\lambda = 1064 \text{ nm}$, with an impinging wave vector lying in the cell midplane $x = 0$. The optical intensity evolution was acquired by collecting the light outscattered by the NLC (through the front slide) with a microscope and a high-resolution camera. Figure 5 shows some typical experimental results for a soliton launched with a small transverse momentum in the waveguide region under the gap. At low powers, the soliton is scattered by and confined within the graded-index profile; as the power increases, the relative effect of the transverse index distribution weakens, eventually letting the self-trapped beam escape the refractive potential. Such trend is better visible in Fig. 6(a), plotting the mean beam trajectory $\langle t \rangle$ vs excitation. Clearly, spatial solitons of small power bounce off the graded-index boundary; as they gain momentum with the nonlinear transverse acceleration, they first propagate along the interface as *surface* solitons [hardly observable in our experiments due to fluctuations, but recently reported by Alfassi and coworkers [19] in thermo-optic glasses at the interface with a linear medium (air)] until they eventually overcome the potential barrier and escape, propagating freely in the surrounding nonlinear region. Considering the coupling losses due to the interfaces and the NLC transition layer at the input, these results are in excellent agreement with the simulations in Fig. 4.

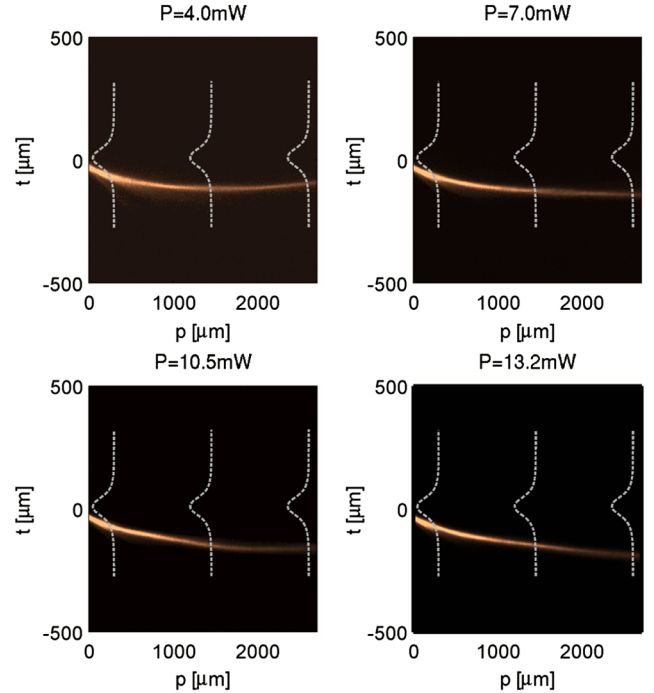


FIG. 5 (color online). Photographs showing the propagation of a soliton in the bias-induced graded-index profile for various optical excitations: at low power, the soliton is trapped in the refractive index well. The dashed bell-shaped curves indicate the transverse distribution of ϕ .

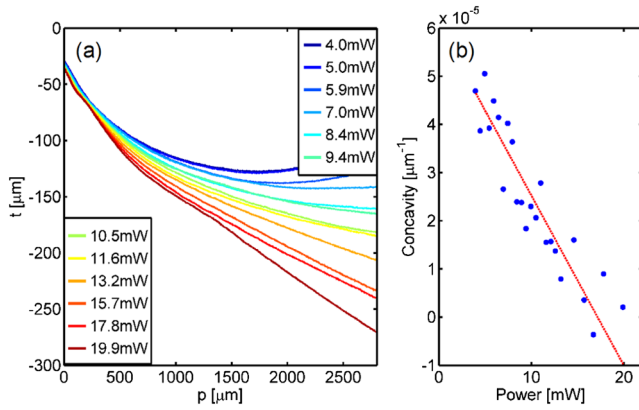


FIG. 6 (color online). Data analysis (averaging over 40 shots): (a) mean soliton path versus input power P . At low power, the solitons are scattered, but for $P > 15$ mW, the initial momentum permits their escape from the voltage-defined potential. (b) Computed concavity of the soliton path between $p = 1700 \mu\text{m}$ and $p = 2000 \mu\text{m}$, i.e., $\partial^2 \langle t \rangle / \partial p^2$, versus input power. The line is a linear fit; standard deviation is $0.6 \cdot 10^{-5} \mu\text{m}^{-1}$.

From the data we extracted, the soliton transverse acceleration after propagation along p for 1.7 mm, i.e., in the proximity of the output, and calculated the mean concavity $\partial^2 \langle t \rangle / \partial p^2$. The results are plotted in Fig. 6(b) versus excitation: as power increases, the concavity approaches zero, demonstrating that at high excitations, the solitons leave the waveguide and escape the potential barrier defined by the external bias.

In conclusion, we presented the first experimental evidence of solitons which, launched within a confining potential with an initial transverse momentum, escape it at high excitations. The phenomenology is well described in terms of the soliton-particle analogy; hence, it establishes yet another strong link between self-confined wave packets and particle physics. Such a demonstration with light-beams, made possible by the optical nonlinearity of nematic liquid crystals and its distribution under bias, is likely to impact all those fields where wave packets are able to self-confine, including matter and electromagnetic waves. We believe that solitons escaping a potential can open novel routes towards soliton-based spatial filtering and limiting as well as new configurations for signal processing.

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